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Performance of Modified Jacobi Sequences with Good Merit Factor

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Abstract: With ideal periodic autocorrelation functions the Quadratic residue and twin prime sequences are well known types of binary sequence. Legendre sequences and modified Jacobi sequences are much larger families of sequences. These sequences do not have ideal autocorrelation functions, but they do exhibit out-of- phase autocorrelation values which are independent of their lengths, and so the longer sequences may be useful. In this project work modified Jacobi sequences are generated for the length < 10000 and k ≤ 30. After generation of the modified Jacobi sequences periodic merit factor has been found out and got the good merit factor 7614 for the length 9991.

Key words: Jacobi sequences, Legendre sequences, Merit factor, Modified Jacobi sequences, Sequences

INTRODUCTION

Merit factor (MF) is defined as the ratio of main lobe energy to side lobes energy [16]. Sequences with good Merit factors are useful for channel estimation, radar, and spread spectrum communication applications. In many areas of communication engineering Binary sequences are extremely useful and many sources have been identified over the years [1, 2]. m-sequences, GMW **(**Gordon-Mills-Welch**)** sequences, Quadratic residue sequences and twin prime sequences have been studied extensively. Unfortunately these sequences are not available in large numbers or for a wide range of sequence lengths. This makes the search for suboptimal but good sequences with a useful activity. Legendre sequences, Jacobi sequences and especially modified Jacobi sequences come under this category. Legendre sequences are introduced in section II. Jacobi sequences defined by use of the Jacobi symbol, known from number theory are introduced in section III. Jacobi sequences are closely related to Legendre sequences. This relation is formulated through the notion of the product of two sequences. For any two sequences of length p and q this product is defined with gcd (p, q) = 1, and the product sequence has length L=p*q. Modified Jacobi sequences, with twin-prime sequences as a special case are also introduced in section IV. Modified Jacobi sequences enhanced the merit factor than the merit factor of the corresponding Jacobi sequence.

LEGENDRE SEQUENCES

For all Prime numbers which are having Lengths L, Legendre or quadratic residue sequences be present[7,10,11,14,15]. They can be constructed using the Legendre symbol

$$\left(\frac{i}{p}\right)$$

This is defined as

$$\left(\frac{i}{p}\right)=\begin{cases}0 & if\ i\, is\, a\, quadratic\, residue \bmod p\\ 1 & otherwise\end{cases} \quad (1)$$

The integer i is a quadratic residue mod p if the equation $x^2 \equiv i \bmod p$ has a solution x which is relatively prime. A Legendre sequence $a=\{a_0\ a_1\ a_2 \ldots\ldots a_{L-1}\}$ is then formed by writing

$$a_i=\left(\frac{i}{p}\right)\quad for\ 0<i<L \qquad (2)$$

and the value of a_0 can be taken either as 0 or 1.As there are exactly (p-1)/2 quadratic residues (QR) and (p-1)/2 quadratic nonresidues (QNR), Legendre sequences are balanced. For example when L=11, the quadratic residues are 1,3,4,5 and 9. The corresponding Legendre sequence is $a=\{1\ 0\ 1\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 1\}$. When L=13, the quadratic residues are 1, 3,4,9,10,12. The corresponding Legendre sequence is $a=\{1\ 0\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 1\ 0\}$. The Legendre sequences for different lengths which are prime, for L<105 are shown in table I.

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Table I: Legendre sequences for L< 105

Class	L	Legendre sequence
1	3	101
2	5	10110
1	7	1001011
1	11	10100011101
2	13	1010011110010
2	17	10010111001110100
1	19	1011000010101111001
1	23	10000101001100110101111
2	29	10110000101110110111010000110
1	31	1001001000011101010001111011011
2	37	1010011010000111011110111000010110010
2	41	10010011000111110101001010111110001100100
1	43	1011010110001000000111010001111011100101001
1	47	10000100001101010001101100100111010100111101111
2	53	10110100100010100011111100110011111100010100010010110
1	59	10100010101101100010000110000011111001111011100100101011101
2	61	1010001110110000001100101101011111101011010011000000110111000010
1	67	1011010110011100001010000001101110100010011111101011110001100101001
1	71	10000001000101100100011100101001011100010110101100011101100101110111111
2	73	1000010100110111010011100010111101100001101111010001110010111011001010000
1	79	1001001100001011010000000100111100111010101010001100001101111110100101111 0011011
1	83	10100110100001110011101010000000101100010011011100101111111010100011000111101001101
2	89	1001001100001111000100011011111101010110010100101001101010111111011000100011 1 100001100100
2	97	100001010010011101011101001011100010011111100110000110011111100100011101001 01 1101011100100101 0000
2	101	1011000110111001001000000011110010110011111010101011010101011111001101001111 0 000000100100111011000110
1	103	1001011000111000000001110100100010001010110111101100101100100001001010111011 10 110100011111110001110010 11

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JACOBI SEQUENCES

Jacobi sequences are closely related to Legendre sequences. This relation is formulated through the notion of the product of two sequences. For any two sequences of length p and q this product is defined with gcd (p, q) = 1.Jacobi sequences [5, 6, 7, 15, 25] be present for all lengths and the product sequence has length L=p*q where both p and q are Prime. They can be constructed using the Jacobi symbol

$$\left[\frac{i}{pq}\right]$$

This is defined as

$$\left[\frac{i}{pq}\right] = \left(\frac{i}{p}\right) \oplus \left(\frac{i}{q}\right) \quad 0 \le i < L \qquad (3)$$

A Jacobi sequence $b = \{b_0 b_1 b_2 ......b_{L-1}\}$ can be formed by writing

$$b_i = \left[\frac{i}{pq}\right] = \left(\frac{i}{p}\right) \oplus \left(\frac{i}{q}\right) \quad 0 \le i < L \quad (4)$$

Thus, b_i = 0 if i, expressed mod p or mod q, is a quadratic residue for both p and q, or is a quadratic nonresidue for both p and q. otherwise b_i = 1.It follows that Jacobi sequences can be constructed from the modulo-2 sum of two Legendre sequences with length p and q, respectively. Consider the case p=5 and q=7 so that L=35. The Legendre sequences of lengths 5 and 7 are 1 0 1 1 0 and 1 0 0 1 0 1 1.Thus the Jacobi sequence of length 35 is formed by term-by-term modulo-2 addition as follows:

10110101101011010110101101011010110

<u>10010111001011100101110010111001011</u>

00100010100000110011011111100011101

MODIFIED JACOBI SEQUENCES

The merit factor of the Jacobi sequence is poor. This merit factor can be improved by using Modified Jacobi sequence. Modified Jacobi sequences enhanced the merit factor than the merit factor of the corresponding Jacobi sequence. The Modified Jacobi sequences of length L= p*q can be defined as follows [5, 6, 7, 15, 28]:

$$b_i = \begin{cases} \left(\frac{i}{p}\right) \oplus \left(\frac{i}{q}\right) & for\ (i, L) = 1 \quad 0 \le i < L \\ 0 & for\ i \equiv 0 \bmod q \\ 1 & otherwise \end{cases} \qquad (5)$$

This modification is equivalent to forcing b_i of the normal Jacobi sequence to be 0 for all i which are multiples of q and to be 1 for all i, other than i=0, which are multiples of p. The replicated versions of the two Legendre sequences of length p and q form the first two components, and expanded versions of the length p sequence and the inverted form of the length q sequence provide the third and fourth components. The third component is made up from the sequence of length p with q-1 0's inserted between each digit and the fourth component is the inverted form of the length q sequence with p-1 0's inserted between each digit. Thus the modified Jacobi sequence can be thought of as a modulo-2 sum of these four component sequence of length p*q. For example, in the case of the length 35 sequence:

10110101101011010110101101011010110

10010111001011100101110010111001011

10000000000000100000010000000000000

<u>10000100001000000000100000000000000</u>

00100110101000010011101111100011101

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Henceforth, it is assumed, without loss of generality, that q > p, so that k = q- p is an even integer. Two distinct classes of modified Jacobi sequences arise depending on the value of k. The Modified Jacobi sequences with k=2 are better known as twin prime sequences [1, 2, 15].

Class 1: k $\equiv$ 2 mod 4. Modified Jacobi sequences in this class have the periodic merit factor can be shown to take the form

$$MF_P = \frac{L^2}{L + 2p(k-2)^2 + k(k-4)^2 - 9} \quad (6)$$

Class 2: k $\equiv 0$ mod 4.Modified Jacobi sequences in this class have the periodic merit factor can be shown to take the form

$$MF_P = \frac{L^2}{5L + 2pk(k-4) + k[(k-4)^2 - 4] - 5} \quad (7)$$

The merit factors of all available modified Jacobi sequences with L<10000 and k$\leq$30 are shown in table II and Fig. 1 shows the variation of the periodic merit factor with sequence length for various values of k.

Table II: Merit factors of all available modified Jacobi sequences with L<10000 and K$\leq$30

K	p	q	L	MF_p	K	p	q	L	MF_p
2	3	5	15	16.07	8	3	11	33	2.43
	7	5	35	36.03		5	13	65	5.74
	11	13	143	144.01		11	19	209	23.74
	17	19	323	324		23	31	713	99.14
	29	31	899	900		29	37	1073	157.46
	41	43	1763	1764		53	61	3233	531.98
	59	61	3599	3600		59	67	3953	661.23
	71	73	5183	5184		71	79	5609	962.6952
	101	103	10403	10404		89	97	8633	1522.4851
4	3	7	21	5.25	10	3	13	39	1.97
	7	11	77	16.29		7	17	119	10.37
	13	17	221	45.06		13	23	299	38.63
	19	23	437	88.25		19	29	551	91.06
	37	41	1517	304.24		31	41	1271	288.99
	43	47	2021	405.04		43	53	2279	638.53
	67	71	4757	952.24		61	71	4331	1501.81
	79	83	6557	1312.2405		73	83	6059	2330.2958
	97	101	9797	1960.2403		79	89	7031	2825.8237
6	5	11	55	13.15	12	5	17	85	3.44
	7	13	91	25.09		7	19	133	6.49
	11	17	187	63.12		11	23	253	15.84
	13	19	247	89.98		17	29	493	37.72
	17	23	391	160.93		19	31	589	47.47
	23	29	667	313.74		29	41	1189	115.61
	31	37	1147	610.77		31	43	1333	133.28

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37	43	1591	907.27	41	53	2173	242.75
41	47	1927	1141.16	47	59	2773	325.77
47	53	2491	1547.4	59	71	4189	531.94
53	59	3127	2021.11	61	73	4453	571.58
61	67	4087	2759.1	67	79	5293	699.6266
67	73	4891	3393.17	71	83	5893	792.6469
73	79	5767	4096.8575	89	101	8989	1287.7242
83	89	7387	5425.3101				
97	103	9991	7614.0413				

K	p	q	L	MF_p	K	p	q	L	MF_p
14	3	17	51	1.13	20	3	23	69	0.65
	5	19	95	3.08		11	31	341	8.44
	17	31	527	40.76		17	37	629	20.76
	23	37	851	81.68		23	43	989	39.6
	29	43	1247	141.49		41	61	2501	142.87
	47	61	2867	461.94		47	67	3149	194.97
	53	67	3551	624.05		53	73	3869	256.76
	59	73	4307	817.55		59	79	4661	328.67
	83	97	8051	1943.8194		83	103	8549	724.3349
	89	103	9167	2322.0195	22	7	21	203	3.1891
16	3	19	57	0.88		19	41	779	26.2724
	7	23	161	4.53		31	53	1643	80.4317
	13	29	377	15.6		37	59	2183	122.4998
	31	47	1457	99.09		61	83	5063	420.353
	37	53	1961	146.51		67	89	5963	533.2378
	43	59	2537	204.77		79	101	7979	813.1043
	67	83	5561	554.5245	24	5	29	145	1.3994
	73	89	6497	672.6639		7	31	217	2.7212
18	5	23	115	2.14		13	37	481	9.4882
	11	29	319	10.75		17	41	697	16.5782
	13	31	403	15.35		23	47	1081	31.5963
	19	37	703	35.43		29	53	1537	52.4691
	23	41	943	54.76		37	61	2257	90.474
	29	47	1363	94.16		43	67	2881	127.3343
	41	59	2419	217.29		47	71	3337	156.1703
	43	61	2623	244.34		59	83	4897	264.6165
	53	71	3763	411.42		73	97	7081	436.0655
	61	79	4819	586.88		79	103	8137	525.3822
	71	89	6319	864.4676					
	79	97	7663	1137.3536					
	83	101	8383	1291.8616					

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K	p	q	L	MF_p	K	p	q	L	MF_p
26	3	29	87	0.4695	30	7	37	259	2.1291
	5	31	155	1.2993		11	41	451	5.3568
	11	37	407	6.457		13	43	559	7.5819
	17	43	731	16.2469		17	47	799	13.3763
	41	67	2747	120.6319		23	53	1219	25.8185
	47	73	3431	167.8084		29	59	1711	43.4002
	53	79	4187	225.2816		31	61	1891	50.5282
	71	97	6887	468.4335		37	67	2479	76.0894
28	3	31	93	0.4217		41	71	2911	96.878
	13	41	533	7.859		43	73	3139	108.4761
	19	47	893	17.3313		53	83	4399	179.5535
	31	59	1829	50.0634		59	89	5251	233.6022
	43	71	3053	104.6482		67	97	6499	320.3996
	61	89	5429	235.5285		71	101	7171	370.5645
	73	101	7373	360.0361		73	103	7519	397.4254

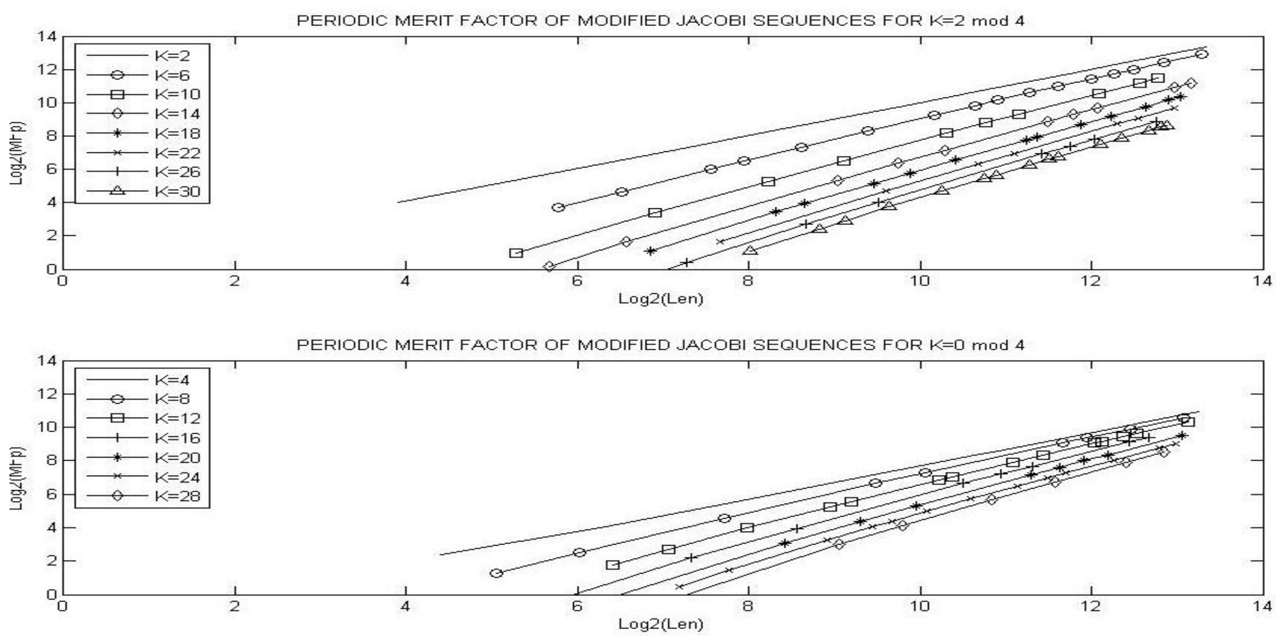

Fig. 1: Periodic merit factor of modified Jacobi sequences for k $\leq$ 30 for K= 2 mod 4 and k= 0 mod 4

From the Fig.1 if K which is difference between two prime factors, increases for different length sequences, the merit factor also increases up to K=20 only and still increases the value of k above K=20 then the merit factor also decreases.

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CONCLUSIONS

Quadratic residue sequences and twin prime sequences are well known types of binary sequences. In this project work we generated modified Jacobi sequences and periodic merit factor has founded. Legendre sequence be present for all lengths L= p, a prime, and Jacobi and modified Jacobi sequences be present for all lengths L = p*q, with p and q both prime. The peak-to-side lobe ratio and the periodic merit factor improve for the longer versions of these sequences.

Design data have been included to enable modified Jacobi sequences with length L< 10000 and with k ≤ 30 to be constructed. From the results we got the good merit factor 7614 for the length 9991 and also got better merit factor for the lengths 7387, 5183, 5767,3599,4891,7031.

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